2026

Four Year Under Graduate Programme (FYUGP)

6th Semester Examination (Under NEP)
(Session 2023-24)

MATHEMATICS (Major)

Paper Code : MTMMJ MC-12

[Probability, Statistics, and Vector Calculus]

Full Marks : 50 | Time : Two Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers
in their own words as far as practicable.

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : $2\times5=10$

(a) Show that the vector $y\hat{j}-z\hat{k}$ is irrotational.

(b) State Green's theorem.

(c) A discrete random variable X has —

$P(X=1)=0.4, P(X=2)=0.6.$

Find $E(X)$.

P.T.O.

(d) If $f(x) = 3x^2, 0 < x < 1$, find $P\left(X < \frac{1}{2}\right)$.

(e) A population has mean 20 and standard deviation 8. For $n = 64$, find the standard error of $\bar{X}$.

(f) If $\phi = xy + yz + zx$, then find $\nabla\phi$.

(g) Given $\text{Cov}(X,Y) = 12, \sigma_X = 3$ and $\sigma_Y = 4$. Find r.

(h) A point P is chosen at random on a line segment AB of length $2a$. Calculate the expected value of the rectangle $AP \cdot PB$.

2. (a) The probability distribution function of a random variable X is given by

$$f(x) = \begin{cases} kx^2 e^{-\frac{x}{2}}, & x > 0 \\ 0, & \text{elsewhere} \end{cases}$$

where k is a suitable constant. Find the value of k.

(b) If X is a binomial (n, p) variate, then show that

$$P(X \le k) = \int_0^q x^{n-k-1}(1-x)^k \, dx \Big/ \int_0^1 x^{n-k-1}(1-x)^k \, dx$$

where $q = 1 - p$ and k is an integer such that $0 \le k \le n-1$. 4+6

3. (a) The probability density function of a random variable X is assumed to be of the form $f(x) = cx^\alpha \ (0 \le x \le 1)$, where c is a constant. If $X_1, X_2, \cdots, X_n$ is a random sample of size n, find the maximum likelihood estimate of α.

(b) The random variables X, Y are connected by the linear relation $aX + bY + c = 0$. Prove that the correlation coefficient between X and Y is -1 if a, b have the same sign and 1 if a, b have opposite signs. 6+4

4. (a) Let $X_1, X_2, \cdots, X_n, \ldots$ be a sequence of random variables and let for X_n, the mean m_n and the standard deviation σ_n exist for all n, then if $\sigma_n \to 0$ as $n \to \infty$, prove that $X_n \to m_n \xrightarrow[in\ p]{} 0$ as $n \to \infty$.

(b) For two random variables X and Y, prove that

$$\text{Var}(aX + bY) = a^2\text{Var}(X) + b^2\text{Var}(Y) + 2ab\,\text{Cov}(X,Y)$$

where a and b are two constants. 5+5

5. (a) If the lines $4x + y = 52$ and $x + y = 32$ be the regression lines of x on y and of y on x respectively, obtain the correlation coefficient between x and y.

P.T.O.

(b) A sample 2.3, –0.2, –0.4, –0.9 is taken from a normal population with variance 9. Find a 95% confidence interval for the population mean. [Given $P(U>1.960) = 0.025$, where U is a normal (0,1) variate.] 5+5

6. (a) Find $\nabla\phi$ if $\phi = \ln|\vec{r}|$.

(b) Find constants a, b and c so that

$$\vec{v} = (-4x - 3y + az)\hat{i} + (bx + 3y + 5z)\hat{j} + (4x + cy + 3z)\hat{k}$$

is irrotational.

(c) Prove that the vector $\vec{A} = 3y^4z^2\hat{i} + 4x^3z^2\hat{j} - 3x^2y^2\hat{k}$ is solenoidal. 3+4+3

7. (a) Verify Stokes' theorem for

$$A = (2x - y)\hat{i} - yz^2\hat{j} - y^2z\hat{k},$$

where S is the upper half of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary. Let R be the projection of S on the xy-plane.

(b) Show that $\left[\vec{a}\times\vec{b}, \vec{b}\times\vec{c}, \vec{c}\times\vec{a}\right] = \left[\vec{a}\vec{b}\vec{c}\right]^2$. 6+4

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Four Year Under Graduate Programme (FYUGP)

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MATHEMATICS (Major)

Paper Code : MTMMJ MC-13

[Complex Analysis]

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : 2×5=10

(a) Write the Cauchy-Riemann equations for a function $f(z) = u(x, y) + iv(x, y)$. Write the equations in polar form also. 1+1

(b) How many fixed points do a bilinear transformation (except the identity map) have? Justify your answer.

P.T.O.

(c) Give an example, with explanation, of a function which is nowhere differentiable on $\mathbb{C}$ but everywhere continuous on $\mathbb{C}$.

(d) If f is a function on a domain $D \subseteq \mathbb{C}$ such that $f'(z) = 0$ for all $z \in D$, prove that f is constant on D.

(e) Show that the function $f(z) = \dfrac{1}{1-e^{z}}$ has a simple pole at $z = 2\pi i$.

(f) Using the Cauchy's integral formula, evaluate the integral $\displaystyle\int_{|z|=1} \frac{\cos 2\pi z}{(2z-1)(z-3)} dz$, where $|z|=1$ is a positively oriented circle.

(g) Find the radius of convergence of the series $\displaystyle\sum_{n=0}^{\infty} (4+5i)^n z^n$.

(h) Determine and classify the nature of the singularity of the function f defined by $f(z) = \dfrac{e^{-z}}{(z-2)^4}$, if any.

2. (a) Let $f(z)=u(x,y)+iv(x,y)$, $z_0 = x_0 + iy_0$, $w_0 = p+iq$.

Prove that $\lim\limits_{z \to z_0} f(z) = w_0$ if and only if

$\lim\limits_{(x,y)\to(x_0,y_0)} u(x,y) = p$ and $\lim\limits_{(x,y)\to(x_0,y_0)} v(x,y) = q$.

(b) Let f be a function defined by

$$f(z) = \begin{cases} \dfrac{x^3 - y^3}{x^2+y^2} + i\dfrac{x^3+y^3}{x^2+y^2}, & z \neq 0 \\ 0, & z = 0. \end{cases}$$

Show that the Cauchy-Riemann equations are satisfied at $z = 0$ but the function f is note differentiable at $z = 0$. 5+5

3. (a) Let f be an analytic function. Prove that

(i) $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$,

(ii) $\left(\dfrac{\partial^2}{\partial x^2} + \dfrac{\partial^2}{\partial y^2}\right)[\operatorname{Re} f(z)]^2 = 2|f'(z)|^2$.

(b) Let f be analytic in a domain D and let γ be a simple closed contour in D, taken in a positive sense. Prove that for all points z interior to γ,

$$f'(z) = \frac{1}{2\pi i}\int \frac{f(\omega)}{(\omega - z)^2} d\omega.$$ (3+3)+4

4. (a) Show that the function u is defined by $u(x,y) = e^x \cos y + y$ is harmonic. Find its harmonic conjugate and construct the corresponding analytic function.

P.T.O.

(b) Find the image of the x-axis under the transformation $w = \frac{i-z}{i+z}$ onto the w-plane.

(2+3)+5

5. (a) Obtain the Laurent's series expansion of the function f defined by $f(z) = \frac{1}{z(1-z)}$ in powers of z.

(b) Find the number of zeros of $z^7 - 4z^3 + z - 1$ inside the circle $|z| = 1$.

5+5

6. (a) Let f and g be analytic within and on a simple closed contour γ and $|f(z)| > |g(z)|$ on γ. Prove that f and $f+g$ have the same number of zeros, counting multiplicities within γ.

(b) Find the singularities of $f(z) = \frac{2z+1}{(z-1)(z-2)^2}$. Also classify the singularities.

5+5

7. (a) Evaluate the following integral :

$$I = \int_{|z|=5} \frac{z+5}{z^2 - 3z - 4} dz .$$

(b) State and prove the Schwarz Lemma, for an analytic function f on $D = \{z : |z| < 1\}$.

5+(1+4)

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MATHEMATICS (Major)

Paper Code : MTMMJ MC-14

[PDE, Integral Equations and Integral Transforms-II]

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks. Candidates are required to give their answers in their own words as far as practicable.

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : 2×5=10

(a) Show that the function $y(x)=\left(1+x^2\right)^{-3/2}$ is a solution of the Volterra integral equation

$$y(x)=\frac{1}{1+x^2}-\int_0^x \frac{t}{1+x^2}\,y(t)\,dt.$$

(b) Convert $\frac{d^2y}{dx^2}+\lambda y(x)=0$ with boundary conditions $y(0)=0, y(1)=0$ to a Fredholm Integral equation.

P.T.O.

(c) Find the eigenvalues of the homogeneous Fredholm Integral equation

$$y(x) = \lambda \int_0^1 e^{x+t} y(t)dt.$$

(d) Find the Fourier cosine transform of e^{-x^2}.

(e) Find the Fourier transform of

$$f(x) = 1 - x^2, \ |x| < 1$$
$$= 0, \quad |x| > 1$$

(f) Find the resolvent kernel of the Volterra integral equation with the kernel, $K(x,t) = x - t$.

(g) Classify the PDE, $3u_{xx} + 2u_{xy} + 5u_{yy} = 0$ as hyperbolic, parabolic or elliptic.

(h) Find the inverse Z-transform of $F(z) = \dfrac{z}{z^2 - 6z + 8}$.

2. (a) Find the eigenvalues and eigenfunctions of the homogeneous integral equation

$$y(x) = \lambda \int_0^1 K(x,t)\, y(t)dt,$$

where $K(x,t) = \begin{cases} t(x+1), & 0 \le x \le t \\ x(t+1), & t \le x \le 1 \end{cases}$

(b) With the help of resolvent kernel, find the solution of the following integral equation

$$y(x) = 1 + x^2 + \int_0^x \frac{1+x^2}{1+t^2} y(t)dt.$$ 6+4

3. (a) Solve the integral equation

$$y(x) = \frac{5x}{6} + \frac{1}{2}\int_0^1 xty(t)\, dt$$

by the method of successive approximation.

(b) Find the iterated kernel for

$$y(x) = f(x) + \lambda \int_a^b K(x,t)\, y(t)dt,$$

where $K(x,t) = x + \sin t, a = -\pi, \ b = \pi.$ 5+5

4. (a) Use Fourier transform to solve the boundary value problem

$$\frac{\partial u}{\partial t} = K\frac{\partial^2 u}{\partial x^2}, u(x,0) = f(x), |u(x,t)| < M, -\infty < x < \infty, t > 0$$

Give a physical interpretation.

(b) Reduce the PDE

$y^2 u_{xx} - 2xy u_{xy} + x^2 u_{yy} = \dfrac{y^2}{x} u_x + \dfrac{x^2}{y} u_y$ to the canonical form. 5+5

P.T.O.

5. (a) Solve the partial differential equation $\frac{\partial^2 u}{\partial t^2} - c^2 \frac{\partial^2 u}{\partial x^2} = 0$, which represents the vibrations of a string of finite length subject to the conditions

$$u(x,0) = f(x),\ 0 \le x \le l$$
$$u_t(x,0) = g(x),\ 0 \le x \le l$$
$$u(0,t) = u(l,t) = 0,\ t > 0$$

by the method of separation of variables.

(b) Using Parseval's identity for Fourier sine and cosine transforms of

$$f(x) = 1,\ 0 \le x \le 1$$
$$= 0,\ x > 1$$

evaluate $\int_0^\infty \left(\frac{1-\cos x}{x}\right)^2 dx$. 6+4

6. (a) If $R(x,t;\lambda)$ be the resolvent kernel of a Fredholm integral equation

$y(x) = f(x) + \lambda \int_a^b K(x,t)\, y(t)\, dt$, prove that the resolvent kernel satisfies the integral equation

$$R(x,t;\lambda) = K(x,t) + \lambda \int_a^b K(x,z)\, R(z,t;\lambda)\, dz.$$

(b) Prove that, for the equation

$$\frac{\partial^2 z}{\partial x \partial y} + \frac{z}{4} = 0$$

the Green's function is

$$w(x,y;\xi,\eta) = J_0\left(\sqrt{(x-\xi)(y-\eta)}\right)$$

where $J_0(z)$ denotes Bessel's function of the first kind of order zero. 5+5

7. (a) Consider the following problem

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0,\ -\infty < x < \infty,\ y > 0$$

with the conditions that $u(x,0) = f(x), -\infty < x < \infty$, $u(x,y)$ is bounded as $y \to \infty$, $u(x,y), \frac{\partial u}{\partial x}$ vanish as $|x| \to \infty$ and f is bounded and continuous in $(-\infty, \infty)$.

Prove that $u(x,y) = \frac{y}{\pi} \int_{-\infty}^{\infty} \frac{f(\xi)}{y^2 + (x-\xi)^2} d\xi$.

P.T.O.

(b) If $Z\{f(n)\} = F(z)$ and $Z\{g(n)\} = G(z)$, then prove that the Z transform of the convolution $f(n) * g(n)$ is given by

$$Z\{f(n) * g(n)\} = Z\{f(n)\}\, Z\{g(n)\}$$

where the convolution is defined by

$$f(n) * g(n) = \sum_{m=0}^{\infty} f(n-m)g(m).$$ 5+5

2026

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MATHEMATICS (Major)

Paper Code : MTMMJ MC-15

[Higher Abstract Algebra]

Full Marks : 50 Time : Two Hours

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Notations and symbols have their usual meanings.

Answer Question No. 1 and any *four* from the rest.

1. Answer any *five* questions : 2×5=10

(a) Let $G=\{e, a, a^2, a^3\}$ be a cyclic group of order 4 and $H=\{e, a^2\}$ be a subgroup of G. Show that H is a characteristic subgroup of G.

(b) Find all non-isomorphic abelian groups of order 8.

(c) Explain why the external direct product $\mathbb{Z}\times\mathbb{Z}$ is not cyclic.

(d) Let G be a non-commutative group of order p^3, p being a prime. Prove that $|Z(G)| = p$.

(e) Show that the polynomial x^3+x+1 is irreducible over $\mathbb{Z}_2$.

(f) Which one among the rings $\mathbb{Z}[x], \mathbb{Z}_6$ and $\mathbb{Z}_{11}[x]$ is a PID? Justify your answer.

(g) Prove that a field $\mathbb{F}$ is a Euclidean domain.

(h) Let G be a group of order 35. Show that G is cyclic.

2. (a) Let G be a group and $Z(G)$ be its centre. Prove that the group Inn(G) of inner automorphisms is isomorphic to the quotient group $G/Z(G)$.

(b) Let G_1 and G_2 be two groups. Prove that the external direct product $G_1 \times G_2$ is commutative if and only if both G_1 and G_2 are commutative. Hence, show $S_3 \times \mathbb{Z}_3$ is not a commutative group.

5+(4+1)

3. (a) Let G be a finite group and p be a prime integer such that $p \mid o(G)$. Show that G has an element of order p.

(b) For a prime p, prove that any group of order p^2 is either cyclic or a direct product of cyclic groups.

5+5

4. (a) Find all the Sylow 3-subgroups of S_4.

(b) Let p and q be two distinct primes with $p < q$ and G be a group of order pq. If p does not divide $q-1$, then show that G is cyclic.

(c) Write down the class equation of A_3.

3+5+2

5. (a) Let $G = A_3 \subset S_3$. For $\sigma = (1\,2\,3)$, find the stabilizer G_σ and the orbit O_σ. Verify the orbit-stabilizer relation $|O_\sigma| = [G : G_\sigma]$.

(b) Show that no group of order 36 is simple.

(c) Give an example of a ring $R[x]$, where degree $(f(x)\cdot g(x)) < \text{degree}(f(x)) + \text{degree}(g(x))$.

(4+1)+4+1

6. (a) Let R be a Euclidean domain with the Euclidean norm δ. Let $u \in R \setminus \{0\}$. Prove that u is a unit in R if and only if $\delta(u) = \delta(1)$.

(b) Show that $\mathbb{Z}_2[x]/\langle x^2+x+1 \rangle$ is a finite field with 4 elements.

(c) Show that a group of order 48 has a normal subgroup of order 16 or 8.

(2+1)+3+4

P.T.O.

7. (a) Show that the ring $\mathbb{Z}[i]=\{a+ib:a,b\in\mathbb{Z}\}$ is a Euclidean domain.

(b) Let F be a field. Prove that $F[x]$ is a PID.

(c) Let G' be a commutator subgroup of a group G. Prove that G is commutative if and only if $G'=\{e\}$. 5+4+1